taken in drawing conclusions from the data of Linke, as drawn in Fig. 2, since it is calculated from experimentally obtained results for the total and pressure drag coefficients, and not by direct measurements of the skin-friction. Figure 3 is a typical plot of the data of Linke for total and pressure drag coefficients. As can be seen from this figure, calculation of the skin-friction drag is very arbitrary because of the scatter in the data.

References

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Turbulent Skin-Friction Coefficient and Momentum Thickness in Adverse Pressure Gradient

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Introduction

Nearly all the available methods for evaluating the skin-friction coefficient are based on the measured mean velocity parameters. Theoretically, it is a function of both local and upstream conditions. If local (surface roughness) conditions are specified, skin friction becomes dependent mainly upon the pressure distribution of the flowfield, and the relation between skin friction and the pressure distribution can be considered unique. The complex nature of the turbulent flow has deterred development of either a theoretical or an empirical relationship between the skin friction and the pressure distribution. This Note is intended to present an empirical relation. The development of momentum thickness is also presented.

Analysis

Sandborn and Liu[1] have given an equation to predict the turbulent boundary-layer separation in a two dimensional flow. It's form is

$$C_p^{(2n-1)/4}[x(dC_p/dx)]^{1/2} = C \tag{1}$$

where C_p is the pressure coefficient, x is the distance measured in the stream direction, C is a constant which depends on n, and n is the $1/n$th power law representation of the velocity distribution at the beginning of the adverse pressure. At such a point, the left-hand term of Eq. (1) is zero and the skin-friction coefficient C_{f0} is assumed known (it can be assumed to be the flat plate condition). As the adverse pressure increases downstream, the position of separation is found at which both terms of Eq. (1) balance, i.e., where the skin-friction coefficient is zero. By this reasoning, the following approximate linear relation may be assumed:

$$C_f/C_{f0} = 1 - \{C_p^{(2n-1)/4}[x(dC_p/dx)]^{1/2}\}/C \tag{2}$$

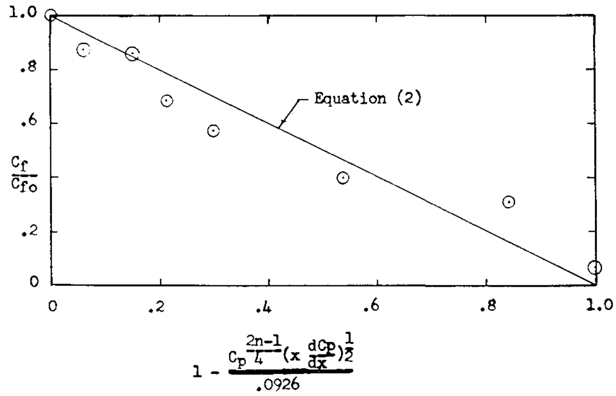

Fig. 1 Variation of skin-friction coefficient.

Figure 1 indicates Schubauer and Klebanoff's[2] experimental data are approximately a straight line.

The momentum integral equation is used to find the momentum thickness development with distance. In a two dimensional flow, its form is

$$d\theta/dx = (H + 2)(\theta/U)(dU/dx) + C_f/2 \tag{3}$$

where θ is the momentum thickness, H is the form factor, and U is the freestream velocity.

Substituting Eq. (2) into Eq. (3) and assuming that H takes a mean value, Eq. (3) can be integrated easily. The final form is

$$\theta = \exp\left(-\int_{x_0}^{x}\frac{H+2}{U}\frac{dU}{dx}dx\right)\left\{\theta_0 + \frac{1}{2}C_{f0} \times \int_{x_0}^{x}\left[1 - \frac{C_p^{(2n-1)/4}(x[dC_p/dx])^{1/2}}{C}\right] \times \exp\left(\int_{x_0}^{x}\frac{H+2}{U}\frac{dU}{dx}dx\right)\right\} \tag{4}$$

where x_0 and θ_0 are, respectively, the length and momentum thickness at the starting position of adverse pressure.

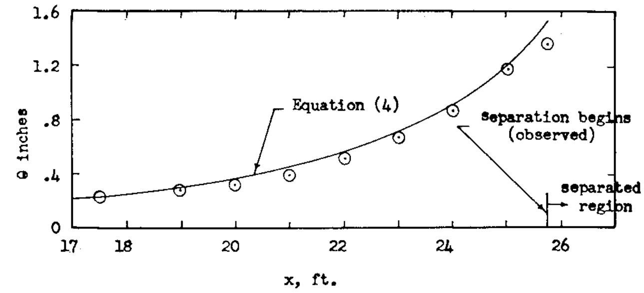

Fig. 2 Development of momentum thickness along the flow.

Using Schubauer and Klebanoff's experimental values, Eq. (4) is plotted in Fig. 2. The measured data are also shown. The result is reasonable.

The mean value of H is found as follows. At the beginning of adverse pressure, H is about 1.30. As separation is approached, H usually assumes a value around 2.6 for an unsteady separation. Hence, $H = 2.0$ is assumed. The constant mean value of H can induce a local maximum error about 15% in H; however, the over-all error in the region of calculation is small.

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